Subject Name: **Geotechnical Engineering-I**

Subject Code: **CE-6003**

Semester: **6th**

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Unit – III

Stress Distribution in Soils and Shear Strength of Soils: Stress distribution beneath loaded areas by Boussinesq and water gaurd's analysis. Newmark's influence chart. Contact pressure distribution.

Mohr – Coulomb's theory of shear failure of soils, Mohr's stress circle, Measurement of shear strength, Shear box test, Triaxial compression test, unconfined compression test, Value shear test, Measurement of pore pressure, pore pressure parameters, critical void ratio, Liquefaction.

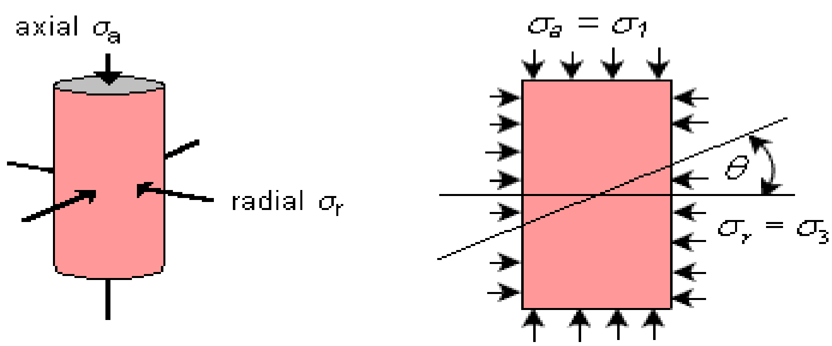
Shear Strength of Soils

Soils consist of individual particles that can slide and roll relative to one another. Shear strength of a soil is equal to the maximum value of shear stress that can be mobilized within a soil mass without failure taking place.

The shear strength of a soil is a function of the stresses applied to it as well as the manner in which these stresses are applied. A knowledge of shear strength of soils is necessary to determine the bearing capacity of foundations, the lateral pressure exerted on retaining walls, and the stability of slopes.

Mohr Circle of Stresses

In soil testing, cylindrical samples are commonly used in which radial and axial stresses act on principal planes. The vertical plane is usually the minor principal plane whereas the horizontal plane is the major principal plane. The radial stress (sr) is the minor principal stress (s3), and the axial stress (sa) is the major principal stress (s1).



To visualise the normal and shear stresses acting on any plane within the soil sample, a graphical representation of stresses called the Mohr circle is obtained by plotting the principal stresses. The sign convention in the construction is to consider compressive stresses as positive and angles measured counter-clockwise also positive.

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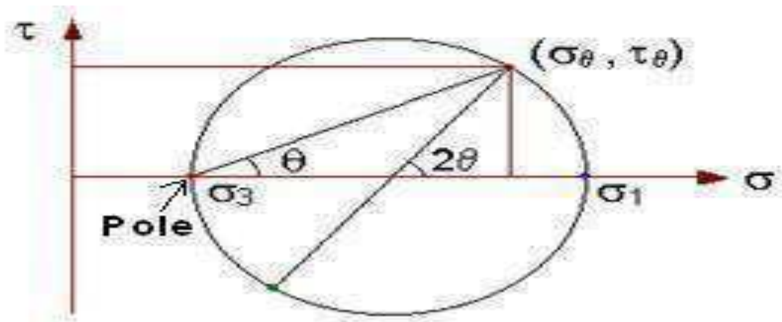


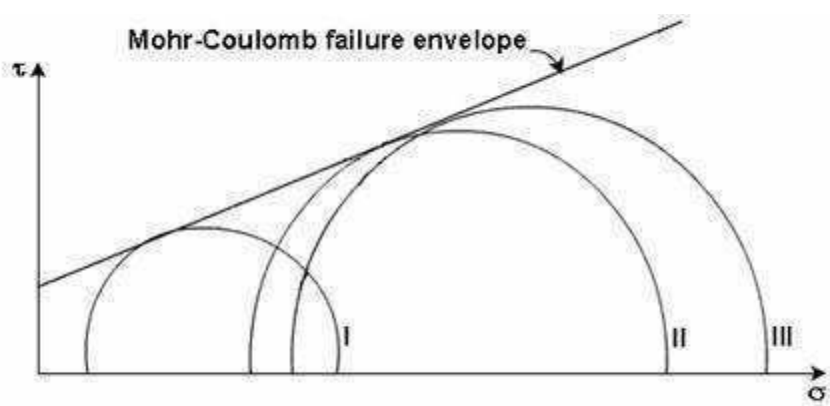
Draw a line inclined at angle with the horizontal through the pole of the Mohr circle so as to intersect the circle. The coordinates of the point of intersection are the normal and shear stresses acting on the plane, which is inclined at angle within the soil sample.

Mohr-Coulomb Failure Criterion

When the soil sample has failed, the shear stress on the failure plane defines the shear strength of the soil. Thus, it is necessary to identify the failure plane. Is it the plane on which the maximum shear stress acts, or is it the plane where the ratio of shear stress to normal stress is the maximum?

For the present, it can be assumed that a failure plane exists and it is possible to apply principal stresses and measure them in the laboratory by conducting a triaxial test. Then, the Mohr circle of stress at failure for the sample can be drawn using the known values of the principal stresses.

If data from several tests, carried out on different samples upto failure is available, a series of Mohr circles can be plotted. It is convenient to show only the upper half of the Mohr circle. A line tangential to the Mohr circles can be drawn, and is called the Mohr-Coulomb failure envelope.



If the stress condition for any other soil sample is represented by a Mohr circle that lies below the failure envelope, every plane within the sample experiences a shear stress which is smaller than the shear strength of the sample. Thus, the point of tangency of the envelope to the Mohr circle at failure gives a clue to the determination of the inclination of the failure plane. The orientation of the failure plane can be finally determined by the pole method.

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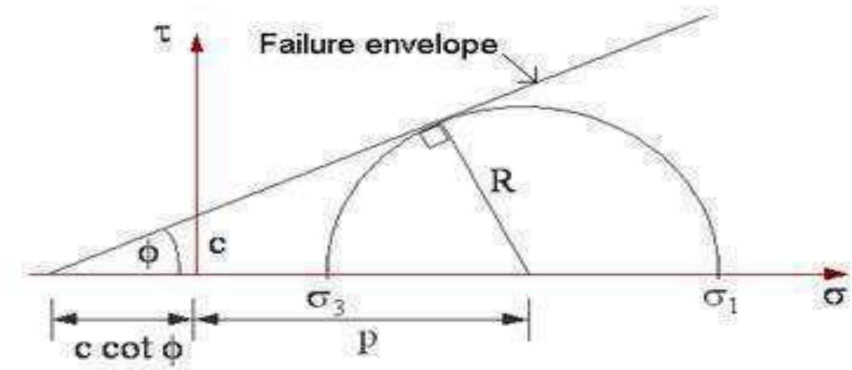

The Mohr-Coulomb failure criterion can be written as the equation for the line that represents the failure envelope. The general equation is

Where

σ_f = shear stress on the failure plane

c = apparent cohesion

= normal stress on the failure plane

f = angle of internal friction

The failure criterion can be expressed in terms of the relationship between the principal stresses. From the geometry of the Mohr circle,

$$\sin\phi = \frac{R}{c.\cot\phi + p} = \frac{\frac{\sigma_1 - \sigma_3}{2}}{c.\cot\phi + \frac{\sigma_1 + \sigma_3}{2}}$$

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

Rearranging,

$$\frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left[\frac{\pi}{4} + \frac{\phi}{2}\right]$$

Normal stress $$\sigma_\theta = \frac{(\sigma_1 + \sigma_3)}{2} + \frac{(\sigma_1 - \sigma_3)}{2}\cos 2\theta$$

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Shear stress $$\tau_\theta = \frac{(\sigma_1 - \sigma_3)}{2}\sin 2\theta$$

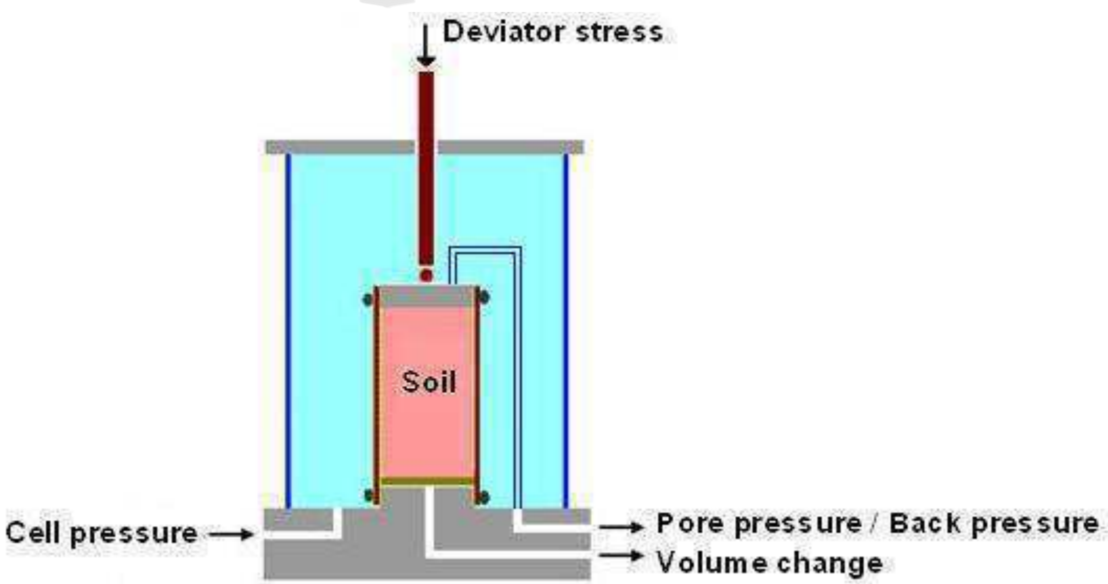
The plane inclined at an angle of to the horizontal has acting on it the maximum shear stress equal to $\frac{\sigma_1 - \sigma_3}{2}$ and the normal stress on this plane is equal to $\frac{\sigma_1 + \sigma_3}{2}$

The plane with the maximum ratio of shear stress to normal stress is inclined at an angle of $45^0 + \frac{\alpha}{2}$ to the horizontal, where a is the slope of the line tangent to the Mohr circle and passing through the origin.

Methods of Shear Strength Determination

Triaxial Test

The triaxial test is carried out in a cell on a cylindrical soil sample having a length to diameter ratio of 2. The usual sizes are 76 mm x 38 mm and 100 mm x 50 mm. Three principal stresses are applied to the soil sample, out of which two are applied water pressure inside the confining cell and are equal. The third principal stress is applied by a loading ram through the top of the cell and is different to the other two principal stresses. A typical triaxial cell is shown.



The soil sample is placed inside a rubber sheath which is sealed to a top cap and bottom pedestal by rubber O-rings. For tests with pore pressure measurement, porous discs are placed at the bottom, and sometimes at the top of the specimen. Filter paper drains may be provided around the outside

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of the specimen in order to speed up the consolidation process. Pore pressure generated inside the specimen during testing can be measured by means of pressure transducers.

The triaxial compression test consists of two stages:

First stage: In this, a soil sample is set in the triaxial cell and confining pressure is then applied.

Second stage: In this, additional axial stress (also called deviator stress) is applied which induces shear stresses in the sample. The axial stress is continuously increased until the sample fails.

During both the stages, the applied stresses, axial strain, and pore water pressure or change in sample volume can be measured.

Test Types

There are several test variations, and those used mostly in practice are:

UU (unconsolidated undrained) test: In this, cell pressure is applied without allowing drainage. Then keeping cell pressure constant, deviator stress is increased to failure without drainage.

CU (consolidated undrained) test: In this, drainage is allowed during cell pressure application. Then without allowing further drainage, deviator stress is increased keeping cell pressure constant.

CD (consolidated drained) test: This is similar to CU test except that as deviator stress is increased, drainage is

permitted. The rate of loading must be slow enough to ensure no excess pore water pressure develops.

In the UU test, if pore water pressure is measured, the test is designated by .

In the CU test, if pore water pressure is measured in the second stage, the test is symbolized as .

Significance of Triaxial Testing

The first stage simulates in the laboratory the in-situ condition that soil at different depths is subjected to different effective stresses. Consolidation will occur if the pore water pressure which develops upon application of confining pressure is allowed to dissipate. Otherwise the effective stress on the soil is the confining pressure (or total stress) minus the pore water pressure which exists in the soil.

During the shearing process, the soil sample experiences axial strain, and either volume change or development of pore water pressure occurs. The magnitude of shear stress acting on different

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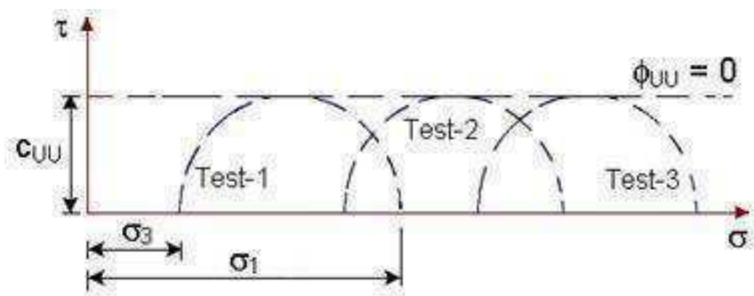
planes in the soil sample is different. When at some strain the sample fails, this limiting shear stress on the failure plane is called the shear strength.

The triaxial test has many advantages over the direct shear test:

- The soil samples are subjected to uniform stresses and strains.
- Different combinations of confining and axial stresses can be applied.
- Drained and undrained tests can be carried out.
- Pore water pressures can be measured in undrained tests.
- The complete stress-strain behaviour can be determined.

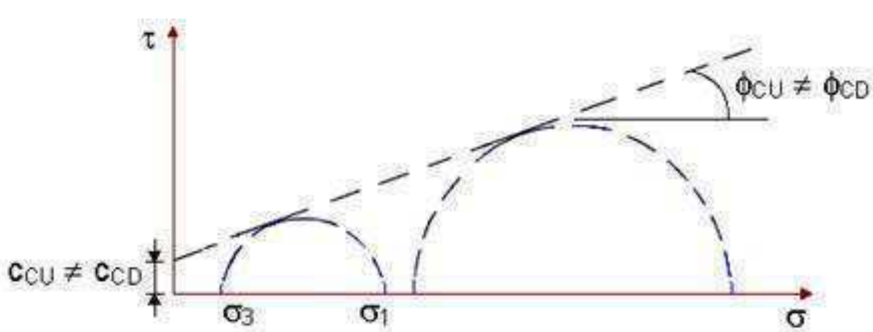
UU Tests:



All Mohr circles for UU test plotted in terms of total stresses have the same diameter.

The failure envelope is a horizontal straight line and hence $\phi_{UU} = 0$ It can be represented by the equation: $\tau_f = c_{UU} = \dfrac{\sigma_1 - \sigma_3}{2}$

CU & CD Tests:



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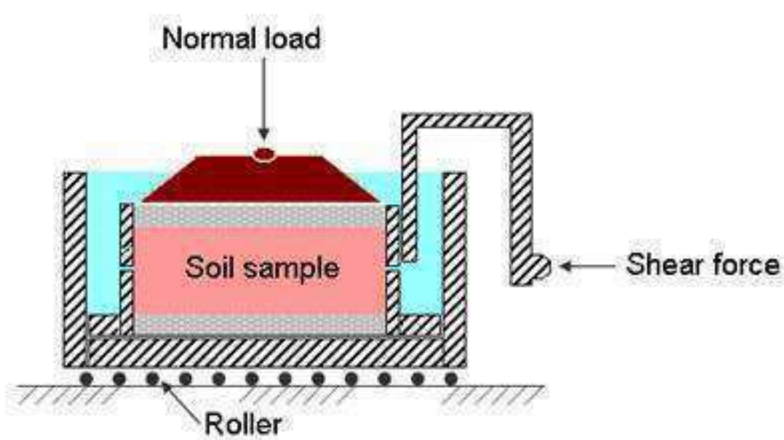
For tests involving drainage in the first stage, when Mohr circles are plotted in terms of total stresses, the diameter increases with the confining pressure. The resulting failure envelope is an inclined line with an intercept on the vertical axis.

It is also observed that cCU [1] cCD and fCU [1] fCD

It can be stated that for identical soil samples tested under different triaxial conditions of UU, CU and CD tests, the failure envelope is not unique.

Direct Shear Test

The test is carried out on a soil sample confined in a metal box of square cross-section which is split horizontally at mid-height. A small clearance is maintained between the two halves of the box.The soil is sheared along a predetermined plane by moving the top half of the box relative to the bottom half. The box is usually square in plan of size 60 mm x 60 mm. A typical shear box is shown.



If the soil sample is fully or partially saturated, perforated metal plates and porous stones are placed below and above the sample to allow free drainage. If the sample is dry, solid metal plates are used. A load normal to the plane of shearing can be applied to the soil sample through the lid of the box.

Tests on sands and gravels can be performed quickly, and are usually performed dry as it is found that water does not significantly affect the drained strength. For clays, the rate of shearing must be chosen to prevent excess pore pressures building up.

As a vertical normal load is applied to the sample, shear stress is gradually applied horizontally, by causing the two halves of the box to move relative to each other. The shear load is measured together with the corresponding shear displacement. The change of thickness of the sample is also measured.

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A number of samples of the soil are tested each under different vertical loads and the value of shear stress at failure is plotted against the normal stress for each test. Provided there is no excess pore water pressure in the soil, the total and effective stresses will be identical. From the stresses at failure, the failure envelope can be obtained.

The test has several advantages:

- It is easy to test sands and gravels.

- Large samples can be tested in large shear boxes, as small samples can give misleading results due to imperfections such as fractures and fissures, or may not be truly representative.

- Samples can be sheared along predetermined planes, when the shear strength along fissures or other selected planes are needed.

The disadvantages of the test include:

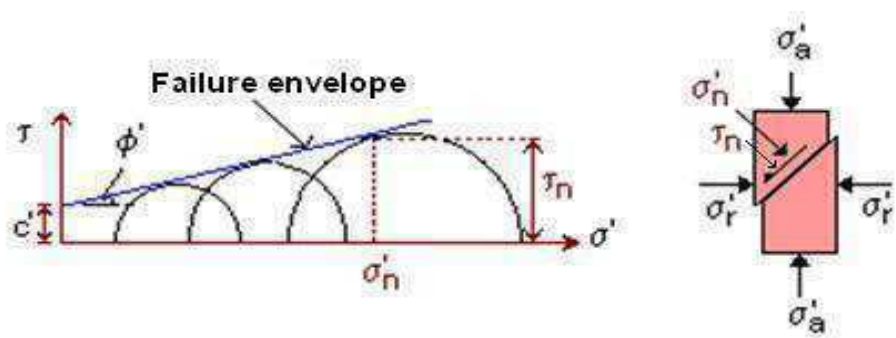
- The failure plane is always horizontal in the test, and this may not be the weakest plane in the sample. Failure of the soil occurs progressively from the edges towards the centre of the sample.

- There is no provision for measuring pore water pressure in the shear box and so it is not possible to determine effective stresses from undrained tests.

- The shear box apparatus cannot give reliable undrained strengths because it is impossible to prevent localised drainage away from the shear plane.

Effective Stress Parameters

If the same triaxial test results of UU, CU and CD tests are plotted in terms of effective stresses taking into consideration the measured pore water pressures, it is observed that all the Mohr circles at failure are tangent to the same failure envelope, indicating that shear strength is a unique function of the effective stress on the failure plane.



This failure envelope is the shear strength envelope which may then be written as

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where c' = cohesion intercept in terms of effective stress

f' = angle of shearing resistance in terms of effective stress

If is the effective stress acting on the rupture plane at failure, is the shear stress on the same plane and is therefore the shear strength.

The relationship between the effective stresses on the failure plane is

$$\sigma_1' = \sigma_3'\left(\frac{1+\sin\phi'}{1-\sin\phi'}\right) + 2c'\sqrt{\frac{1+\sin\phi'}{1-\sin\phi'}}$$

Pore Water Pressure Parameters

The difference between the total and effective stresses is simply the pore water pressure u. Consequently, the total and effective stress Mohr circles have the same diameter and are only separated along the s - axis by the magnitude of the pore water pressure.

It is easy to construct a series of total stress Mohr Circles but the inferred total stress parameters have no relevance to actual soil behaviour. In principle, the effective strength parameters are necessary to check the stability against failure for any soil construction in the field. To do this, the pore water pressure in the ground under the changed loading conditions must be known and in general they are not.

In an undrained triaxial test with pore pressure measurement, this is possible and the effective stresses can then be determined. Alternatively, in drained tests, the loading rate can be made sufficiently slow so as to allow the dissipation of all excess pore water pressure. For low permeability soils, the drainage will require longer times.

In undrained tests, the general expression relating total pore water pressure developed and changes in applied stresses for both the stages is: Du = Du1 + Du2 = B.Ds3+ B.A.(Ds1 - Ds3) = B[Ds3+ A(Ds1 - Ds3)]

where Du1 = pore water pressure developed in the first stage during application of confining stress Ds3,

Du2 = pore water pressure developed in the second stage during application of deviator stress (Ds1 - Ds3), and B and A are Skempton's pore water pressure parameters.

Parameter B is a function of the degree of saturation of the soil (= 1 for saturated soils, and = 0 for dry soils). Parameter A is also not constant, and it varies with the over-consolidaton ratio of the soil and also with the magnitude of deviator stress. The value of A at failure is necessary in plotting the effective stress Mohr circles.

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Consider the behaviour of saturated soil samples in undrained triaxial tests. In the first stage, increasing the cell pressure without allowing drainage has the effect of increasing the pore water pressure by the same amount.

Thus, there is no change in the effective stress. During the second shearing stage, the change in pore water pressure can be either positive or negative.

For UU tests on saturated soils, pore water pressure is not dissipated in both the stages (i.e., Du = Du1 + Du2).

For CU tests on saturated soils, pore water pressure is not dissipated in the second stage only (i.e., Du = Du2).

Stress-Strain Behaviour of Sands

Sands are usually sheared under drained conditions as they have relatively higher permeability. This behaviour can be investigated in direct shear or triaxial tests. The two most important parameters governing their behaviour are the relative density (ID) and the magnitude of the effective stress (s¢). The relative density is usually defined in percentage as

$$I_D = \frac{e_{\max} - e}{e_{\max} - e_{\min}} x100$$

where emax and emin are the maximum and minimum void ratios that can be determined from standard tests in the laboratory, and e is the current void ratio. This expression can be re-written in terms of dry density as

$$I_D = \left(\frac{\gamma_d - \gamma_{d\min}}{\gamma_{d\max} - \gamma_{d\min}} \right) x \frac{\gamma_{d\max}}{\gamma_d} x100$$

where gdmax and gdmin are the maximum and minimum dry densities, and gd is the current dry density. Sand is generally referred to as dense if ID > 65% and loose if < 35%.

The influence of relative density on the behaviour of saturated sand can be seen from the plots of CD tests performed at the same effective confining stress. There would be no induced pore water pressures existing in the samples.

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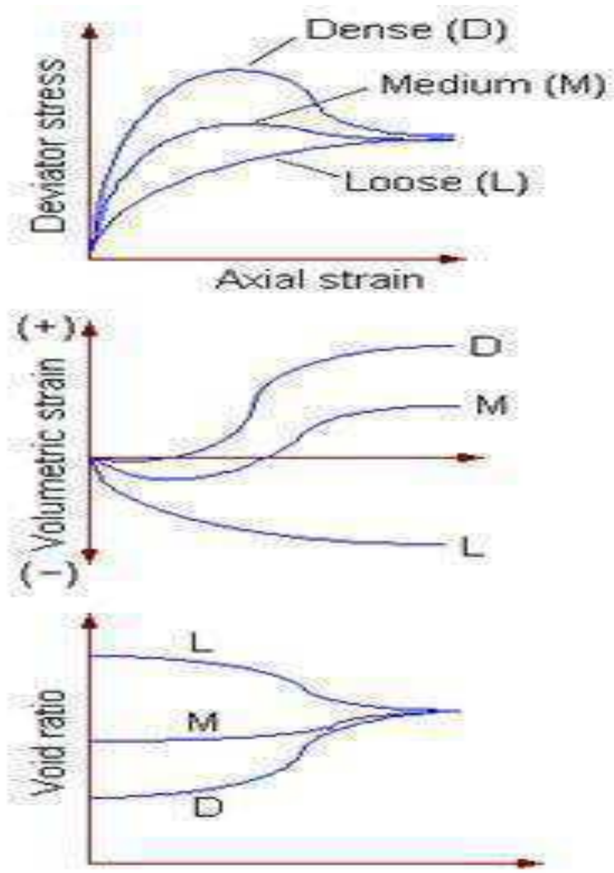

For the dense sand sample, the deviator stress reaches a peak at a low value of axial strain and then drops down, whereas for the loose sand sample, the deviator stress builds up gradually with axial strain. The behaviour of the medium sample is in between. The following observations can be made:

• All samples approach the same ultimate conditions of shear stress and void ratio, irrespective of the initial density. The denser sample attains higher peak angle of shearing resistance in between. Initially dense samples expand or dilate when sheared, and initially loose samples compress.

Worked Examples

Example 1: A UU test is carried out on a saturated normally consolidated clay sample at a confining pressure of 3 kg/cm2.The deviator stress at failure is 1 kg/cm2.

(a) Determine its total stress strength parameters.

(b) If another identical sample is tested at a confining pressure of 4 kg/cm2, what will be the vertical axial stress at failure?

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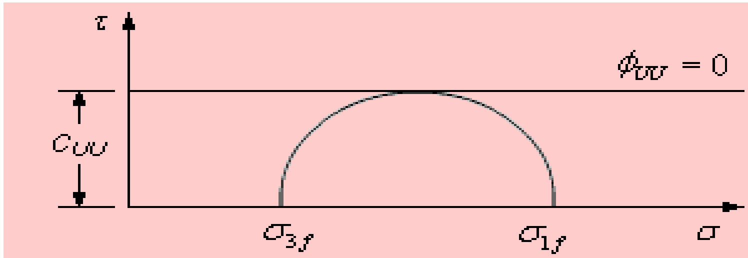

$\sigma_{3f} = 3\, kg/cm^2$

$\sigma_{1f} - \sigma_{3f} = 1\, kg/cm^2$

(a) From the plot, note that fUU = 0 and

$$c_{UU} = \frac{\sigma_{1f} - \sigma_{3f}}{2} = 0.5\, kg/cm^2$$

(b) $\sigma_{3f} = 4\, kg/cm^2$

UU tests on identical samples yield the same failure deviator stress $(\sigma_{1f} - \sigma_{3f})$ **at all confining pressures. Therefore, the vertical axial stress at failure,** $\sigma_{1f} = 4 + 1 = 5\, kg/cm^2$

Example 2: Results of tests conducted on two saturated clay samples are given. Determine the shear strength parameters.

	Sample1	**Sample2**
Confining pressure -----	**4.8 kg/cm2**	**6.3 kg/cm2**
Axial stress at failure --------------	**6.8 kg/cm2**	**9.3 kg/cm2**
Pore water pressure at failure ---	**3.8 kg/cm2**	**4.8 kg/cm2**

Solution:

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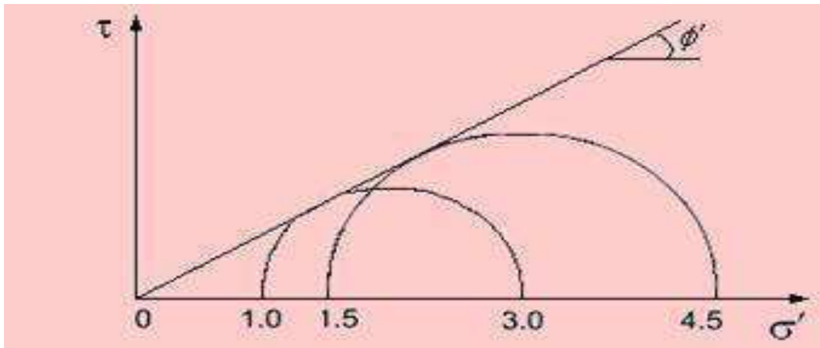

For sample 1:

$$\sigma'_{3f} = \sigma_{3f} - u_f = 4.8 - 3.8 = 1.0\, kg/cm^2$$
$$\sigma'_{1f} = \sigma_{1f} - u_f = 6.8 - 3.8 = 3.0\, kg/cm^2$$

For sample 2:

$$\sigma'_{3f} = \sigma_{3f} - u_f = 6.3 - 4.8 = 1.5\, kg/cm^2$$
$$\sigma'_{1f} = \sigma_{1f} - u_f = 9.3 - 4.8 = 4.5\, kg/cm^2$$

From the plot, one can obtain

$$c' \approx 0$$
$$\phi' = 30°$$

Newmark's Influence Chart - Stress determination at a depth z in soil

Hi,Howhaveyou been?

Well, we can find out the stresses for the rectangular, circular or strip loading using the boussinesq's equation, but Newmark's influence charts were prepared to calculate the stress below an irregular shaped uniformly loaded areas. In such cases, these charts are extremely useful.

Newmark's charts are based on the vertical stress at a point P below the center of a circular uniformly loaded area.

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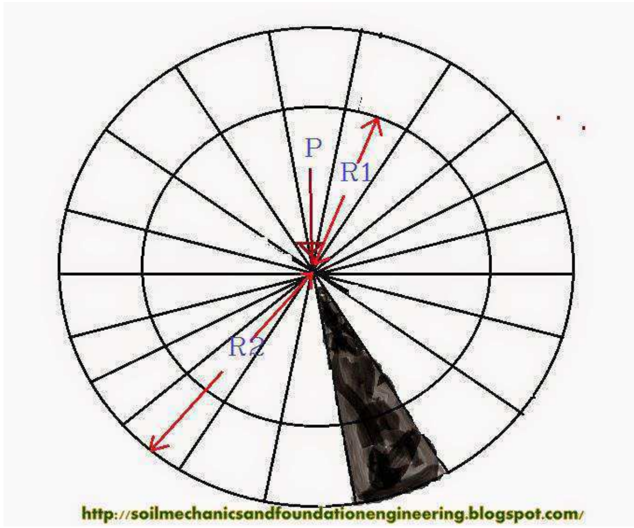

Concentric circles for R1 and R2 (Newmark's Chart)

Consider a circle of radius R1, divided into 20 equal sectors. The vertical stress at a point below the center of the circle at depth z due to a uniform load on one sector will be equal to 1/20th of that due to the load on the entire circle.

$$\sigma_z = \frac{1}{20}q\left\{1-\left\{\frac{1}{1+[R_1/z]^2}\right\}\right\}$$

If in above equation, stress is given as arbitrary value, say 0.005.q, we can solve the equation to get,

R1/z = 0.270

Thus every twentieth sector of the circle, with radius R1 equal to 0.270.z would produce a stress of 0.005.q at a depth z from its center.

Now consider the second concentric circle with radius R2, in this circle, suppose the area of the strip excluding the area under the strip of radius R1, produces a stress at point P, equal to 0.005.q, then the total stress at P would be equal to 2*0.005.q.

Putting these values in above equation, R2/z = 0.40, In other words, the radius of the second circle, should be equal to 0.40.q.

Similarly, the values of the Radii of 3rd to 9th circle can be determined. The values are 0.60.z, 0.77.z, 0.92.z, 1.11.z, 1.39.z and 1.91.z. The radii of 9.5th circle is 2.54.z.

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The radii for the 10th circle when calculated comes to infinity.

Therefore the 10th circle can not be drawn.

Remember each enclosed area in between the different circles and the sector lines has an influence of 0.005.q. There are 20 sectors and 9 circles, and thus the Newmark's influence chart is ready.

- **Procedure to determine the value of the Stress at a given point P at a death z**

1. **Measure the depth z and precisely locate the point P where the stress is to be determined.**
2. **Plot the plan of the loaded area with a scale of z equal to unit length of the chart (AB).**
3. **Place the plan on the chart in such a manner that the point P, at which the stress is to be determined is placed directly below the center of the chart.**
4. **Count the number of elements(M) of the chart enclosed by the plan of the loaded area.**

The formula which can be used to determine the stress at depth z, due to the given loaded area can be given as, Stress = (IF).q.M

Here IF = Influence Factor, which we have taken equal to 0.005

q = pressure intensity at top.

M = Number of elements of the chart covered by the prepared plan.

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